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# Vortex Pairing as a Model for Jet Noise Generation

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**An attempt to use a simplified vortex model to study jet noise is reported. Special considerations are given to relate the vortex ring properties with jet flow parameters, such as jet velocity, nozzle radius, and jet-structure Strouhal number. Results show good agreement with existing low-Mach-number turbulence sound theories and experimental observations of an initially laminar jet. They also give new insight into some well-established jet noise phenomena.**

## Nomenclature

| | |
|---|---|
| $A$ | = constant depending on core vorticity distribution |
| $a_0$ | = ambient speed of sound |
| $C$ | = initial vortex ring separation |
| $D$ | = nozzle diameter |
| $f$, $f_d$ | = frequency and directivity function, respectively |
| $I$ | = sound intensity |
| $k_1, k_2, k_3$ | = proportionality constants |
| $L$ | = stroke length |
| $M, M_c$ | = jet Mach number and convective Mach number of quadrupole source, respectively |
| $n$ | = unit normal in the direction of vortex ring propagation |
| $p$, $p_{\text{peak}}$, $p_{\text{rms}}$ | = far-field pressure, maximum far-field pressure, and root-mean-square value, respectively |
| $R, R_o, R_m$ | = initial vortex ring radius, far-field observer distance, and mid-width of a jet, respectively |
| $St_D$ | = Strouhal number, $fD/W$ |
| $s$ | = infinitesimal arc on the vortex ring |
| $T_{ij}$ | = Lighthill's stress tensor |
| $t$ | = time |
| $t_\omega$ | = vortex ring formation interval |
| $U_\omega$ | = total propagation speed for an isolated vortex ring |
| $u$ | = velocity |
| $V_j$ | = jet velocity |
| $W$ | = fluid velocity at the mouth of nozzle |
| $\boldsymbol{x}, x$ | = far-field displacement and its magnitude, respectively |
| $\boldsymbol{y}, y$ | = near-field displacement and its magnitude, respectively |
| $z$ | = downstream distance from nozzle exit |
| $\alpha$ | = background flow parameter |
| $\Gamma$ | = vortex ring circulation |
| $\theta$ | = emission angle |
| $\xi$ | = axial displacement of vortex ring |
| $\rho_0$ | = density of ambient fluid |
| $\sigma, \sigma_c, \sigma_e$ | = radial coordinate, core radius, and effective core radius, respectively |
| $\omega$ | = vorticity |

## Introduction

LIGHTHILL[1,2] showed mathematically that the far-field noise from turbulent flow could be estimated by solving the inhomogeneous wave equation

$$\frac{1}{a_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where $T_{ij}$ in the low-Mach-number limit is $T_{ij} \approx \rho_0 u_i u_j$. In this case the sound from turbulent flow is generated by the fluctuating fluid momentum transfer within the flow. The solution of Eq. (1), that is, the far-field pressure fluctuations at a distance $\boldsymbol{x}$ from the flow, can be obtained by integrating the second time derivative of $T_{ij}$ over the whole volume of the flow:

$$p \sim \frac{1}{4\pi a_0^2}\frac{x_i x_j}{x^3}\int \frac{\partial^2}{\partial t^2} T_{ij}(\boldsymbol{y}, t - |\boldsymbol{x} - \boldsymbol{y}|/a_0)\, \mathrm{d}\boldsymbol{y}^3 \tag{2}$$

Lighthill's theory was extended for application to jet noise by Ffowcs Williams.[3] He showed that the far-field intensity of subsonic jet flow can be written in dimensional form as

$$I \propto \frac{V_j^8 D^2 f_d(\theta)}{R_o^2(1 - M_c \cos\theta)^5} \tag{3}$$

where $M_c$ is the convection speed of the turbulent eddies divided by the speed of sound in the external fluid and $f_d(\theta)$ is the directivity function of the unconvected quadrupole sources. Equation (3) implies that the eddy convection speed does not affect the sound intensity at a 90-deg emission angle. The theories of Lighthill[1,2] and of Ffowcs Williams[3] also suggest that the turbulent jet flow can be thought of as comprising many compact convected quadrupole radiators. However, the volume integral in Eq. (2), in general, cannot be calculated without substantial simplification of the source region. The assumption of isotropic turbulence in jet shear flow, as employed by Ribner[4] to evaluate the volume integral, does not seem to be realistic because the large-scale structures in a shear layer, especially those formed before the fully developed region, possess vorticity in one particular direction and thus do not usually result in isotropic turbulence.[5] The generality of their approaches[1–4] also made the Lighthill theory difficult to apply in practice. In addition, the simplified acoustic analogy of Lighthill[1] does not explain all of the features found in the experimental studies of subsonic jet noise[6,7] and does not clarify the physical mechanism of jet noise generation.

Inspired by the experimental results of Mollo-Christensen,[8] which show the existence of substantial correlation between near-field pressure fluctuations measured across the jet diameter just outside the turbulent zone, Crow[9] formulated the line-antenna model for jet noise calculation. In this model, the jet instability is modeled as an axisymmetric sine wave with a Gaussian amplitude axial distribution. However, though the model gives some results that are consistent with experimental observations, it has not been explored sufficiently for application to the jet noise problem.[10] There are also a number of basically different jet noise models (e.g., Tester and Morfey[11]), but they are seldom related to the jet structure dynamics, which should be the major source of noise.

Powell[12] showed that for low Mach numbers the right-hand side of Eq. (1) can be rewritten as $-\nabla \cdot (\omega \times u)$. This theory relates the

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sound generation mechanism in a subsonic jet to the dynamics of the vorticity in the jet. The discovery of organized structures in the jet shear layer by many researchers, such as Davies et al.[13] and Crow and Champagne,[14] suggest that the organized structures play an important role in the generation of jet noise. Becker and Massaro[15] and Winant and Browand[16] observed that vortex pairing is the basic mechanism of jet development at low Reynolds number. The experimental results of Kibens[17] further suggest that a highly ordered vortex-pairing cascade, under controlled acoustic excitation, generates acoustic waves that propagate to the far field. The above-mentioned experimental results, together with the analytical results of Möhring,[18] which illustrate quadrupole far-field sound pattern from the pairing of two closely packed thin-core vortex rings, contribute to a general belief that vortex pairing is the dominant sound source in a subsonic jet. On the other hand, Bridges and Hussain[19] argue that the dominant source of turbulent jet noise is the breakdown of coherent vortex rings close to the end of the potential core, the proposed cut-and-connect mechanism of vortex ring breakdown by Bridges and Hussain[19] and Hussain[20] is hard to visualize in a subsonic jet. Thus the theory of breakdown noise generation is still not well established. Also from the results of Kibens[17] and Bridges and Hussain,[19] vortex breakdown noise is important only for unexcited initially turbulent jets. However, the results of Bridges and Hussain[19] illustrate clearly that the noise from an initially turbulent jet, which they suggested comes mainly from vortex breakdown, is considerably lower than that from an initially laminar jet, which comes from vortex ring pairing. This shows the importance of pairing noise studies in jet noise research.

Some experiments have attempted to relate sound generation with vortex motions in jets. Laufer and Yen[21] found that the sound source within a low-Mach-number jet is compact and is in the area where the instability is moving at high speed. The more recent work of Tang and Ko[22] shows that the sound generation in a subsonic jet is related to the accelerating motions of the flow structures during the pairing process. Thus the experimental results mentioned above tend to support vortex pairing as the dominant source of subsonic jet noise. Recently, using the contour dynamics method, Tang and Ko[23] studied the sound from vortex ring interactions, but did not explicitly relate the pairing noise with jet noise.

Because the vortex pairing process can be easily handled analytically, it may be useful for the prediction of jet noise if it is found to be the dominant source of noise. To clarify this point, an analysis of the sound field that it produces and an investigation into its consistency with existing turbulence sound theories and experimental results are desirable. This paper documents a numerical analysis of the vortex pairing noise. The extent of its consistency with experiments and other theories is also discussed.

## Models for Vortex Formation and Pairing Noise

Vortex pairing in the present investigation refers to the mutual slip-through motion of two coaxial inviscid vortex rings.[24] This type of vortex pairing was visualized by Yamada and Matsui.[25] The motions of the vortex rings are first calculated and then the far-field noise is computed using the formula of Möhring.[18]

In the interaction of several vortex rings, the ring cores are assumed to be circular and the vorticity within them is constant. The Biot–Savart law of velocity induction is used to estimate the vortex ring velocity[26]:

$$V_i = -\frac{1}{4\pi}\sum_{i\neq j}\Gamma_j\int\frac{(\boldsymbol{y}_i-\boldsymbol{y}_j)}{|\boldsymbol{y}_i-\boldsymbol{y}_j|^3}\times\frac{d\boldsymbol{y}_j}{ds_j}\,ds_j + \frac{\Gamma_i}{4\pi R_i}\left[\log\left(\frac{8R_i}{\sigma_i}\right)-\frac{1}{2}+A\right] \tag{4}$$

where $\boldsymbol{y}$ is a point on the vortex rings. The subscript $i$ refers to the vortex ring under consideration and $j$ to the other vortex rings. The positions of the vortex rings are obtained by integrating Eq. (4) with respect to time, using the fourth-order Runge–Kutta method. The far-field pressure fluctuation $p$ is computed by Möhring's formula[18]:

$$p = \frac{\rho_0}{4a_0^2x^3}\sum_i\left(\frac{d^3\Gamma_i R_i^2\xi_i}{dt^3}\right)\boldsymbol{x}\times\left(\boldsymbol{nn}-\frac{1}{3}\right)\times\boldsymbol{x} \tag{5}$$

A similar vortex-pairing noise calculation has been done by Kambe and Minota,[26] but they did not address the mechanism of sound generation in relation to jet noise.

The parameters that affect the vortex ring motions, and thus the sound generation, are the vortex ring circulation, ring radius, core size, and initial separation of the interacting vortex rings. For comparison between vortex-pairing noise and jet noise, the relationship of these parameters to the jet velocity, nozzle diameter, and Strouhal number must be found. Saffman[27] showed that the vortex ring generated by pushing a piston with a speed $W$ and a stroke length $L$ inside a sharp-edged circular nozzle of diameter $D$ has a radius $R$, core radius $\sigma_c$, and circulation $\Gamma$, which are approximately given by the following formulas:

$$R = 0.5\left(D + k_1L^{\frac{2}{3}}D^{\frac{1}{3}}\right) \tag{6a}$$

$$\sigma_c = k_2L^{\frac{2}{3}}D^{\frac{1}{3}} \tag{6b}$$

$$\Gamma = k_3WLD^{\frac{2}{3}} \tag{6c}$$

These results were verified by the experimental results of a number of researchers such as Maxworthy.[28] Since the speed of piston movement should be the same as the fluid velocity at the exit of the nozzle for incompressible flow, $W$ is viewed as the jet velocity in the present analysis. Let $t_\omega$ be the vortex ring-formation interval, then

$$t_\omega = 1/f = D/St_DW \tag{7}$$

The stroke length, therefore, can be written in the form

$$L = Wt_\omega = D/St_D \tag{8}$$

In the present model, the vortex rings are initially separated by an axial distance $C$ (see Fig. 1). Assuming the speed of the vortex ring propagation is that given by Lamb[29] and including a background flow velocity of $\alpha W$, where $\alpha$ is a nonzero constant, the total vortex ring propagation velocity is

$$U_\omega = \frac{\Gamma}{4\pi R}\left[\log\left(\frac{8R}{\sigma_c}\right)-\frac{1}{2}+A\right]+\alpha W \tag{9}$$

Thus

$$C = U_\omega t_\omega = \left\{\frac{\Gamma}{4\pi R}\left[\log\left(\frac{8R}{\sigma_c}\right)-\frac{1}{2}+A\right]+\alpha W\right\}\frac{D}{St_DW}$$
$$= \frac{k_3D}{2\pi\left(St_D^{\frac{2}{3}}+k_1\right)St_D^{\frac{2}{3}}}\left[\log\left(\frac{4\left(St_D^{\frac{2}{3}}+k_1\right)}{k_2}\right)-\frac{1}{2}+A\right]$$
$$+\frac{\alpha D}{St_D} \tag{10}$$

Equation (10) gives the relationship between the initial core separation in the present model and the Strouhal number of jet structure.

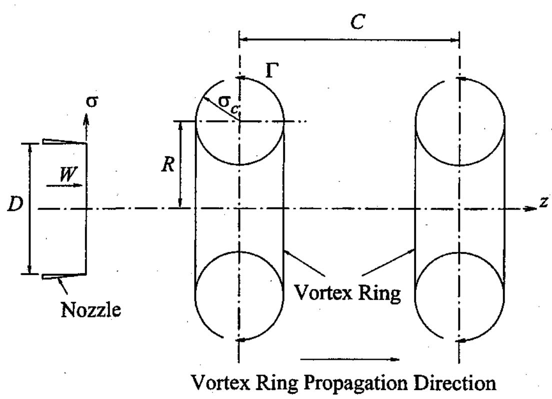


**Fig. 1 Schematic diagram of vortex ring model.**

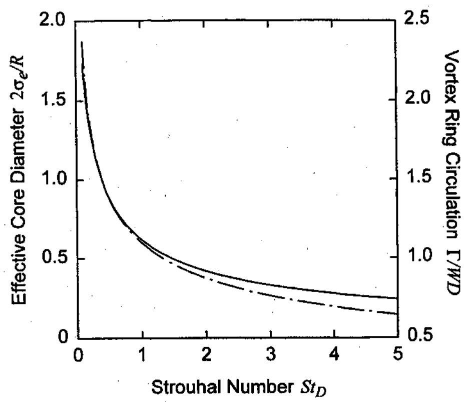


**Fig. 2 Variations of effective core diameter and vortex ring circulation with Strouhal number: $W/W_0 = 1, D/D_0 = 1, \alpha = 0.5$; ——, $2\sigma_e/R$; and —·—, $\Gamma/WD$.**

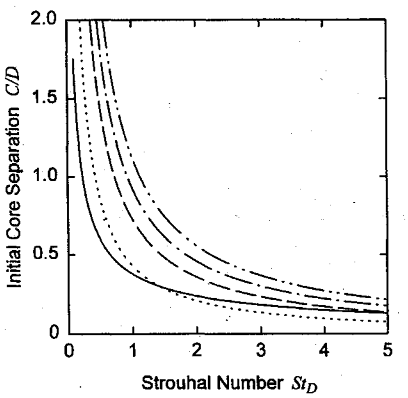


**Fig. 3 Variations of initial core separation with Strouhal number for different $\alpha$: $W/W_0 = 1, D/D_0 = 1$; ——, $2\sigma_e/R$; · · ·, $\alpha = 0$; — —, $\alpha = 0.3$; —·—, $\alpha = 0.5$; and —··—, $\alpha = 0.7$.**

The proportionality constants $k_1$, $k_2$, and $k_3$ are assumed to be 0.22, 0.4, and 1.1, respectively.[27] For the vorticity distribution, $A = 1$, as suggested by Saffman.[27] Equation (6) can now be rewritten as

$$R = 0.5D\left(1 + 0.22St_D^{-\frac{2}{3}}\right) \tag{11a}$$

$$\sigma_c = 0.4DSt_D^{-\frac{2}{3}} \tag{11b}$$

$$\Gamma = 1.1WDSt_D^{-\frac{1}{3}} \tag{11c}$$

Equations (10) and (11) show that the ratio $\sigma_c/R$ depends solely on Strouhal number. Figure 2 illustrates the variation of the ratio of effective core radius $\sigma_e$ to vortex ring radius with Strouhal number. The effective core radius corresponds to the case of constant vorticity within the core[27]:

$$\sigma_e = \sigma_c e^{\frac{1}{4}-A} \tag{12}$$

Thus the higher the Strouhal number, the smaller the core radius relative to that of the vortex ring. This agrees with experimental observations of, for example, Becker and Massaro.[15] The initial vortices, which appear close to the lip of a circular jet, have high Strouhal numbers but small core size. Also, as shown in Fig. 2, at high Strouhal numbers, less ejected fluid rolls up to form the vortex ring and weaker vortex rings are expected.

The initial core separation $C/D$ depends not only on Strouhal number but also on $\alpha$, which in the present study denotes the effect of background jet mean flow. The unknown $\alpha$ affects the Strouhal number range of the present study: $C/D$ has to be larger than the effective core diameter $2\sigma_e$ to avoid overlapping of vortex cores. Certainly, as the value of $\alpha$ increases, the initial core separation increases and the chance of core overlapping at high Strouhal numbers decreases (see Fig. 3). Note that with an $\alpha$ of 0.3, core overlapping can be avoided for $St_D < 4$. The effect of $\alpha$ is discussed further below.

After establishing the relationship between the parameters in the present model with those used to describe jet flow, the effects of $R$, $C$, and $\Gamma$ on the sound radiated by vortex pairing can now be compared with the results of subsonic jet noise measurements. The aim of the present investigation is to study trends rather than to compare actual values. Thus the jet velocity $W$ and nozzle diameter $D$ are normalized by arbitrary constants $W_0$ and $D_0$, respectively. $W_0$ and $D_0$ are so chosen that a jet of velocity $W_0$ issuing from a nozzle of diameter $D_0$ gives a far-field pressure amplitude $p_0$, which is used to normalize the far-field pressure fluctuations unless otherwise stated.

## Results and Discussion

The present theory of vortex ring pairing breaks down when the core size becomes large, and so the investigation will be restricted to $St_D > 0.2$. According to Crighton and Gaster,[30] the mid radius of a single circular jet $R_m$, which is defined as the radius at which the mean axial velocity equals half the jet centerline velocity, is related to the local shear-layer momentum thickness $\theta_m$ and downstream distance $z$ by

$$R_m/\theta_m = 100/(3z/R_m + 4) \tag{13}$$

For very small $z$, where the initial shear-layer vortices are formed, $R_m/\theta_m \approx D/2\theta_m \approx 25$. On the basis of the experimental circular jet results of Hussain,[20] which show that the initial shear-layer mode has a Strouhal number based on local momentum thickness of $f\theta_m/W \approx 0.017$, the corresponding Strouhal number based on jet diameter $D$, $fD/W$, is expected to be 0.85 in an unforced jet. Allowing for the presence of harmonics in a forced jet,[17] the Strouhal number range investigated in the present study is extended to $St_D = 4$, which corresponds to about the fourth harmonic of the most unstable shear-layer mode instability $fD/W = 0.85$.

Equation (5) shows that vortex pairing generates a quadrupole sound field, which agrees with the results of low-Mach-number jet noise measurements of, for example, Bridges and Hussain.[19] Figure 4 shows the far-field pressure fluctuations at $St_D = 0.5$ and 2 for $W/W_0 = 1$, $D/D_0 = 1$, and $\alpha = 0.5$. The abscissa is the normalized time of flight, which denotes the time elapsed after the start of the pairing interaction and is numerically equal to the far-field retarded time. Peaks or sound pulses are generated at the instants when one vortex ring slips through the other. The patterns of the variation suggest that the root-mean-square value of the far-field sound pressure depends significantly on the Strouhal number. This issue is discussed later.

As shown in Fig. 5, the variation of far-field sound pressure amplitude with jet velocity $W$ obeys the eighth-power law of sound intensity radiation for low-Mach-number unsteady turbulent flow at a fixed Strouhal number.[1–3] The far-field sound pressure amplitude also varies with the fourth power of vortex ring circulation $\Gamma$, because of the linear relationship between circulation $\Gamma$ and jet velocity $W$ at fixed $St_D$ [Eq. (11c)]. Figure 5 also shows that the far-field sound pressure amplitude increases with Strouhal number

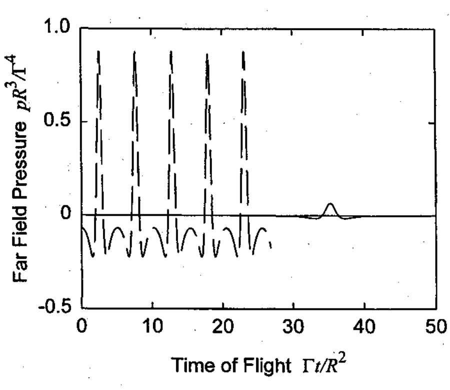


**Fig. 4 Time variations of far-field pressure fluctuations: $W/W_0 = 1, D/D_0 = 1, \alpha = 0.5$; ——, $St_D = 0.5$; and — —, $St_D = 2$.**

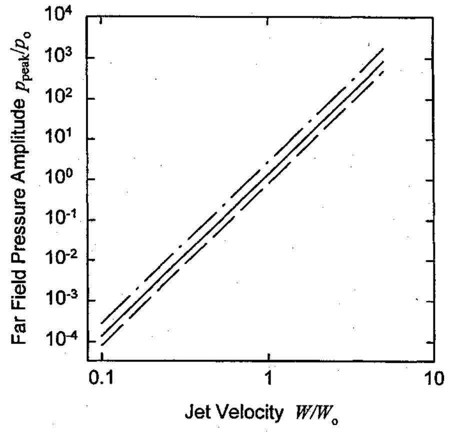


**Fig. 5 Variations of maximum far-field pressure with jet velocity for different Strouhal numbers: $D/D_0 = 1$, $\alpha = 0.5$; — —, $St_D = 0.5$; ——, $St_D = 1$; and —·—, $St_D = 2$.**

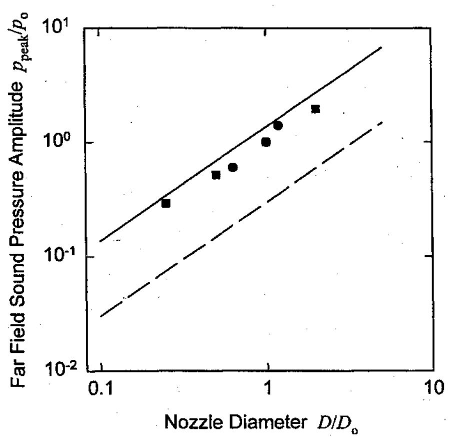


**Fig. 6 Variations of maximum far field pressure and root-mean-square pressure with nozzle diameter: $W/W_0 = 1$, $\alpha = 0.5$, $St_D = 1$; ——, $p_{peak}$; — —, $p_{rms}$; ■, Lush[6]; and ●, Ahuja and Bushell.[7]**

for a fixed combination of jet velocity $W$ and nozzle diameter $D$. Equations (11) and (12) illustrate that the vortex ring circulation $\Gamma$, vortex ring radius $R$, vortex core size $\sigma_c$, separation $C/D$, and $\sigma_c/R$ decrease with increasing Strouhal number; thus the above observation implies that the reduction in the separation $C/D$ at higher Strouhal number results in more unsteady vortex ring motion. This effect overrides the decrease in vortex ring circulation, giving rise to stronger sound radiation. Therefore for a fixed jet velocity the pairing process of the initial shear-layer vortices in an initially laminar jet results in higher far-field sound pressure level than that generated by vortex ring pairing in the jet column. This may be the reason for the absence of subharmonics in the far-field pressure spectrum of the initially laminar jet of Bridges and Hussain.[19] The root-mean-square far-field pressure intensities $p_{rms}$ also conform to the eighth-power law of Lighthill[1] and thus its dependence on $W$ is not presented.

The nozzle diameter $D$ affects the vortex ring circulation and the radius for a fixed Strouhal number and thus affects the sound generated from vortex ring pairing. For a fixed combination of $W$ and $St_D$, the amplitude and root-mean-square values of the far-field sound pressure are proportional to $D$, as shown from the slope of the straight lines in Fig. 6. This means that the intensity is proportional to $D^2$ and is consistent with the findings in jet noise theory and experiments (e.g., Lighthill,[1] Lush,[6] and Ahuja and Bushell[7]). The present results also agree with the findings of Ffowcs Williams[3] at a 90-deg emission angle. The normalized far-field sound pressure is independent of $D$ and is not presented.

Equations (11) and (12) show that a change in the Strouhal number alters the initial conditions of the interacting system. As shown in Fig. 4 for a low Strouhal number of 0.5, the sound pressure level is significant only at or near the instant when one vortex ring slips through the other and is negligible the rest of the time. For a Strouhal

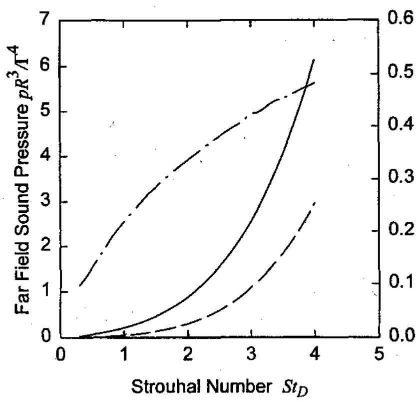


**Fig. 7 Variations of $p_{peak}$, $p_{rms}$, and $p_{rms}/p_{peak}$ with Strouhal number: $W/W_0 = 1$, $D/D_0 = 1$, $\alpha = 0.5$; ——, $p_{peak}$; — —, $p_{rms}$; and —·—, $p_{rms}/p_{peak}$.**

number of 2, however, the radiated sound pressure is significant throughout the interaction period. Figure 7 shows the variations of the normalized $p_{peak}$, $p_{rms}$ and their ratio with Strouhal number for $W/W_0 = 1$, $D/D_0 = 1$, and $\alpha = 0.5$. The $p_{peak}$ and $p_{rms}$ in Fig. 7 are normalized by $\Gamma^4/R^3$. Though the difference between the $p_{peak}$ and $p_{rms}$ increases rapidly at high $St_D$, the increase of the ratio $p_{rms}/p_{peak}$ with $St_D$ reveals that at higher Strouhal numbers, the contribution of the sound pressure generated at the instant of vortex ring slip-through in the average radiated sound intensity becomes less significant. This is consistent with the results shown in Fig. 4. The separation between the vortex rings increases as $St_D$ decreases (see Fig. 3). This implies that the mutual induction strength between the vortex rings also increases with $St_D$. Thus the motion of vortex rings is more unsteady at high $St_D$, regardless of how close the vortex ring slip-through instant, and the sound generated during this period then becomes significant at high $St_D$, giving the physical explanation for the observation in Fig. 7. Figure 7 also indicates that the rate of increase of $p_{rms}/p_{peak}$ decreases as $St_D$ increases. It further implies that the far-field sound pressure level increases with $St_D$, though the circulation and the impulse of vortex rings decrease with $St_D$ [Eq. (11)]. The findings provide an answer to the experimental excited jet results of Kibens[17] that a high near-field pressure fluctuation level does not correspond to strong far-field radiation. The near-field pressure measured by a microphone is the aerodynamic pressure associated with the jet fluid motion in the entrainment region of the excited jet. Its magnitude, therefore, is related more directly to the strength of the flow structures than to the sound power produced by their interaction.

Though the time variations of the far-field pressure fluctuations have periodic patterns (see Fig. 4), their spectra are broadband with maximum spectral density at the frequency of the pairing motion, as shown in Fig. 8. The lower the Strouhal number, the more broadband is the far-field noise spectrum. It is also observed that these spectra contain peaks at the Strouhal number of vortex ring formation. This suggests that the appearance of a broadband low-frequency peak at $St_D \approx 0.4$ and the absence of a distinct peak at the initial shear-layer vortex pairing frequency in the far-field pressure spectra of the initially laminar jet of Bridges and Hussain[19] may be a result of vortex ring pairing in the jet column, though they believe that the peak comes from the breakdown of vortex ring.

So far, the present vortex model produces results that are consistent with the power-law characteristics of existing jet noise theories at low Mach number and 90-deg emission angle. In the low-Mach-number case, the convection speed of the quadrupole sources does not affect the sound power and thus the radiated far-field amplitude. However, there is not a distinct division between low and high Mach numbers in the present model, even though the theory is based on the assumption of low-Mach-number flow. A discussion of the effect of the quadrupole convection speed on the far-field noise in the present model follows.

The speeds of the vortex rings vary during their interaction,[22] but the centroid of the pairing system convects steadily downstream with constant speed equal to that of the isolated vortex ring for any

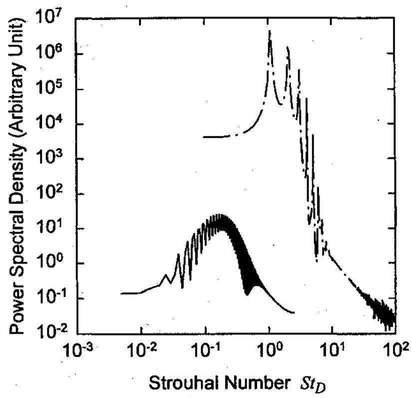


**Fig. 8 Spectra of far-field pressure fluctuations: $W/W_0 = 1, D/D_0 = 1, \alpha = 0.5$; ——, $St_D = 0.3$; and — —, $St_D = 1$.**

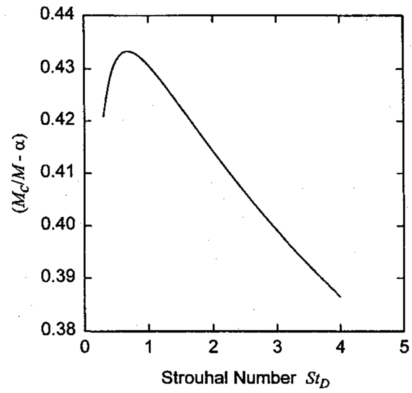


**Fig. 9 Effect of Strouhal number on quadrupole source convection speed.**

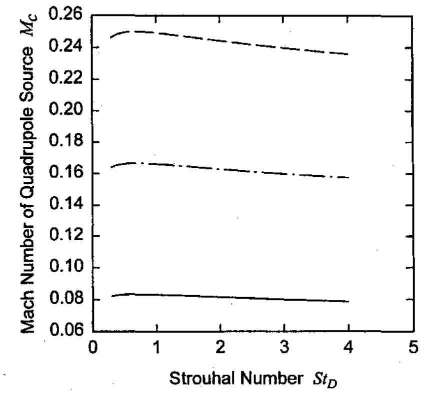


**Fig. 10 Effect of jet velocity on Mach number of convecting quadrupole: $\alpha = 0.4$; ——, $M = 0.1$; —·—, $M = 0.2$; and — —, $M = 0.3$.**

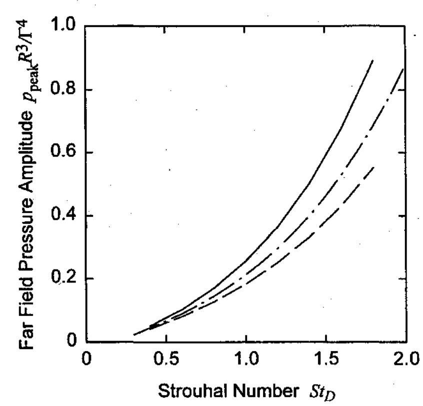


**Fig. 11 Variations of maximum far-field pressure with Strouhal number for different $\alpha$: $W/W_0 = 1, D/D_0 = 1$; ——, $\alpha = 0.4$; —·—, $\alpha = 0.5$; and — —, $\alpha = 0.6$.**

combination of $W$, $D$, and $St_D$. Therefore the convection speed of the quadrupole source is the speed of an isolated vortex ring. From Eq. (9), the convection Mach number of the quadrupole source $M_c$ is given by

$$M_c = \frac{U_\omega}{a_0}\frac{\Gamma}{4\pi R a_0}\left[\log\left(\frac{8R}{\sigma_c}\right) - \frac{1}{2} + A\right] + \alpha M$$

$$= M\left\{\frac{1.1 St_D^{\frac{1}{2}}}{2\pi\left(St_D^{\frac{2}{3}} + 0.22\right)} \times \left[\log\left(\frac{4\left(St_D^{\frac{2}{3}} + 0.22\right)}{0.4}\right) + \frac{1}{2}\right] + \alpha\right\} \tag{14}$$

where $M = W/a_0$. $M_c/M$ depends on $St_D$ and the mean flow parameter $\alpha$. Figure 9 shows the variation of $M_c/M$ with $St_D$ for $W/W_0 = 1$, $D/D_0 = 1$. Because $\alpha$ is assumed to be a constant and should not affect the trend of the variation, it is excluded from the calculation. The present model suggests that the quadrupole source convects at the highest speed when $St_D \approx 0.7$. However, this number depends significantly on the values of $k_1$ and $k_2$. Because these values are not known exactly, errors may occur in the estimation of the $St_D$ for the maximum source convection speed. However, $k_1, k_2$, and $k_3$ do not seem to affect the trend of the variation. Although the present model suggests a smaller $M_c$ at higher $St_D$, the variation of $M_c$ is only about 10% (see Fig. 9). For a fixed $M$, this supports the assumption of constant $M_c$ employed in jet noise studies by many researchers, such as Ahuja and Bushell[7] and Tester and Morfey,[11] and the experimental results of almost constant $M_c$ at different $St_D$ of Davies et al.[13] and Petersen.[31]

The present model shows that when the jet Mach number $M$ increases, a change in $St_D$ results in bigger change in $M_c$ [Eq. (14)]. As Eq. (14) shows, $(M_c/M - \alpha)$ depends only on $St_D$ and decreases for $St_D > 0.7$ [this value depends on $k_1, k_2$, and $k_3$ introduced in Eq. (6)], and the rate of increase in $M_c$ with $M$ declines as $St_D$ increases for $St_D > 0.7$ (Fig. 10). This indicates that the difference in $M_c$ for different-frequency quadrupole sources increases with $M$ for a fixed $\alpha$. Therefore such increase in $M_c$ is higher at lower $St_D$ as Mach number $M$ increases. This increase in $M_c$ results in an increase in the propagation speed of the convecting quadrupole source toward the far-field observer at emission angle $\theta$ other than 90 deg. The Doppler factor $(1 - M_c \cos\theta)^{-1}$, as shown in Eq. (3) and discussed by Lighthill[1] and Ffowcs Williams,[3] increases with $M_c$ so that as $M$ increases, lower-Strouhal-number quadrupole sources in the jet mixing layer tend to produce somewhat higher sound pressure levels than the higher frequency sources. Vortex pairing in the jet column mode then becomes more important than that in the shear-layer mode as a source of sound at higher jet velocity. This seems to explain the loss in significance of the shear-layer mode pairing noise ($St_D \approx 2.7$) in an initially laminar circular jet as $M$ increases from 0.15 to 0.35.[19] For a fixed combination of $W$, $D$, and $St_D$, the increase in $\alpha$, and thus in $C/D$, reduces the mutual induction strength, resulting in a lower far-field noise level (see Fig. 11). Though the latter increases with $St_D$, its rate of increase decreases as $\alpha$ increases. This shows that the faster the axial propagation speed of the vortex ring system, and thus that of the quadrupole source in jet mixing layer, the lower the sound power level. This effect becomes significant at high jet Mach number.

The change in Strouhal number $St_D$ affects not only the initial vortex ring separation $C/D$ and $M_c$ but also the vortex ring circulation and radius. The effect of $M_c$ on the far-field sound radiation is thus hard to visualize from the change of $St_D$. By changing the mean flow parameter $\alpha$, which does not affect the vortex ring properties, any change in the far-field sound pressure is the result solely of the change in $C/D$ or $M_c$. Keeping $W/W_0 = 1$, $D/D_0 = 1$, and $St_D = 1$, the variations of $p_{peak}$ and $p_{rms}$ with $\alpha$ are shown in Fig. 12a. Note that $p_{peak} \propto \alpha^{-0.8}$ but $p_{rms} \propto \alpha^{-1.25}$ for $0.3 \le \alpha \le 0.9$. The

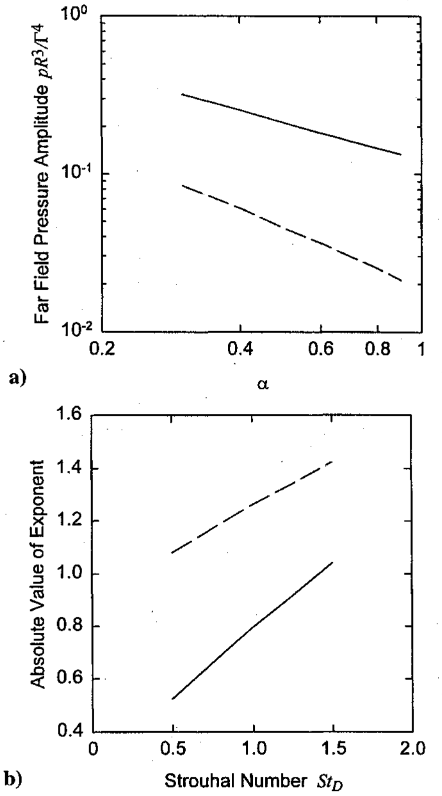


**Fig. 12 a) Variations of far-field pressure amplitude with $\alpha$: $W/W_0 = 1, D/D_0 = 1, St_D = 1$. b) Variations of power-law exponent with Strouhal number: $W/W_0 = 1, D/D_0 = 1$; ——, $p_{\text{peak}}$; and — —, $p_{\text{rms}}$.**

absolute values of the exponents increase with $St_D$ (see Fig. 12b). This suggests that the same pair of vortex rings radiate less sound energy if they are propagating in a region of higher mean flow. The present model suggests that the presence of a coflowing stream external to a jet can reduce jet noise caused by the increase of mean velocity in the mixing layer.[32,33] The higher the $St_D$ of the jet structures, the higher the noise attenuation. This seems to be consistent with the results of coaxial jet noise studies.[34] In addition, because the background mean flow is $\alpha W$, which is related to jet velocity, the combined effect of $\alpha$ and $W$ may result in an approximately fifth to sixth velocity power law of sound radiation, if $\alpha$ varies with Strouhal number or $M$.[11] The convection speeds of vortical structures within jet mixing layers vary from 0.4 $V_j$ to approximately $V_j$, depending on Strouhal number, suggesting a complicated dependence of the present hypothetical $\alpha$ on Strouhal number and jet velocity. Further investigation is needed. The power-law relationship between $\alpha$ and far-field sound amplitude shown in Fig. 12a and Eq. (14) indicates that there is no power-law relationship between the latter and the quadrupole-source convection speed.

## Conclusions

The present paper reports the use of a simplified vortex-pairing noise model to account for the results of jet noise measurements. This model of mutual slip-through vortex rings takes into account the formation process of the jet structures and their subsequent pairing motions. The vortex ring circulation, core size, and radius in the model are related to jet velocity, nozzle diameter, and flow-structure Strouhal number by empirical formulas.

The consistency of the present model with existing turbulence sound theory is discussed. It is found that the present model illustrates the eighth-power law of low-Mach-number jet noise. Also, the amplitude of the sound calculated increases linearly with nozzle diameter, which again is consistent with existing jet sound theories at 90-deg emission angle or at very low Mach numbers. Besides, the present model suggests higher sound power radiation at higher Strouhal number for a fixed jet velocity, though the corresponding vortex rings become weaker. This agrees with the experimental results for excited jets. As the jet velocity increases, the lower-Strouhal-number sources become important in the generation of sound. In addition, the far-field sound pressure power spectrum shows a broadband peak at the Strouhal number of vortex ring formation. Despite the lack of experimental support, the findings suggest that the spectral peak found in the jet noise spectrum may not come solely from the frequency halving process. The broadband spectral peak at the preferred jet column mode found in laminar jet, therefore, also may come from vortex ring pairing instead of from the breakdown of the vortex ring.

The speed of the quadrupole source, which is the convection speed of the whole vortex pairing system in the present model, does not change significantly with Strouhal number. It is also found that the background mean flow reduces the sound power radiated. This shows the possibility of sound reduction by introducing an external coflowing stream to a jet. The lower noise level of coaxial jets may be the outcome of this effect. The present model also illustrates the possibility that far-field sound intensity varies with the fifth or sixth power of jet velocity.

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